The Impact of Spatial Uncertainties in the Magnetic Reluctivity on the Field Quality of a Combined Function Magnet

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In iron dominated magnets the field homogeneity inside the aperture is influenced by the magnetic behaviuor law, expressed via the magnetic reluctivity, of the yoke. In practice this law is uncertain due to manufacturing imperfections. In this paper the uncertain magnetic reluctivity is modeled as a random field and it is discretized by using the truncated Karhunen-Loève expansion (KLE). Among all other expansions the KLE yields a minimal truncation error in the mean square sense. The suggested stochastic model is used to study the statistics of the individual multipole coefficients, that heavily influence the beam dynamics, in a combined function magnet.

***Index Terms*—Combined function magnet, field quality, Karhunen-Loève expansion, uncertainty quantification.**

I. Introduction

Combined function magnets are used in particle accelerators, as at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, Germany, to control both horizontal and vertical deflection of charged particles. The field homogeneity, or the field quality, inside the aperture is an important design requirement during the R&D phase and it is influenced by both, the superconducting coils and the magnetic yoke. The field quality is described by a set of Fourier coefficients known as field harmonics or multipole coefficients. These coefficients are used to expand the radial component of the magnetic flux density at a given reference radius inside the aperture.

Numerical simulations are required to determine the field quality. The magnetic behaviour law enters as an input parameter in those simulations. In practice this law is uncertain due to the imperfection of the manufacturing process. In order to obtain reliable simulation results the uncertainties in this law have to be taken into account. In particular it is impotant to quantify the impact of the uncertain material law on the higher multipole coefficients that are known to deteriorate the field quality. The uncertain material properties should be modeled as random fields and for the purpose of numerical simulation those random fields must be discretized, i.e., represented via a finite number of random variables. For computationally efficient simulations it is required that the number of random variables is as small as possible. In this context we propose to use the truncated Karhunen-Loève expansion (KLE). When the KLE is truncated after M terms the error is smaller compared to any other $M-$term expansion in the mean square sense [1].

In order to use the KLE one needs to know the covariance function of the random field that is deduced from measurement data. So far the KLE has been applied for the stochastic modeling of a nonlinear, homogeneous and isotropic magnetic materials in [2] and the covariance function has been deduced from actual measurements that are already presented in [3]. The impact of uncertainties in nonlinear magnetic materials on the field quality has been studied in [4] with the Brauer model. The KLE has been used for the stochastic modeling of spatial uncertainties in the magnetic reluctivity in [5]. In contrast to the previous work in this paper the impact of spatial uncertainties in the magnetic reluctivity on the field quality are considered. For simplicity the uncertainties in the nonlinear law are not taken into account. The combination of both will part of a future work.

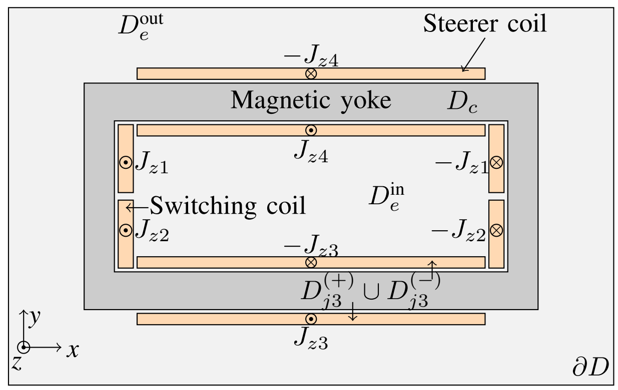

Fig. 1: 2D cross section of a combined function magnet

II. Governing equations

The 2D cross section of the combined function magnet is shown in Fig. (1). The air, coil and the magnetic yoke domains are denoted as D_e, D_j and D_c, respectively. The air domain consists of the inner part D_e^{in} and the outer part D_e^{out}, i.e., $\overline{D}_e = \overline{D}_e^{\text{out}} \cup \overline{D}_e^{\text{in}}$. The p-th coil domain is denoted as $\overline{D}_{jp} = \overline{D}_{jp}^{(+)} \cup \overline{D}_{jp}^{(-)}$ where $p \in \{1, ..., 4\}$. The total computational domain is $\overline{D} = \overline{D}_e \cup \overline{D}_j \cup \overline{D}_c$ and its boundary is denoted as ∂D. The magnetic reluctivity and the current density are defined as follows:

$$\nu(\vec{x}, \theta) = \begin{cases} \nu_e & \text{in } D_e, \\ \nu_j & \text{in } D_j, \\ \nu_c(\vec{x}, \theta) & \text{in } D_c, \end{cases} \quad J_{zp} = \begin{cases} \dfrac{N_{tp} I_p}{S_p} & \text{in } D_{jp}^{(+)}, \\ \dfrac{-N_{tp} I_p}{S_p} & \text{in } D_{jp}^{(-)}, \end{cases} \tag{1}$$

where N_{tp} is number of turns, S_p is the cross sectional area of the p-th coil, I_p is the current and θ is an outcome of a random event. The governing stochastic magnetostatic partial differential equation is given as

$$\begin{aligned} -\nabla \cdot (\nu(\vec{x}, \theta) \nabla A_z(\vec{x}, \theta)) = J_z(\vec{x}) \quad &\text{in} \quad D, \\ A_z(\vec{x}, \theta) = 0 \quad &\text{on} \quad \partial D, \end{aligned} \tag{2}$$

where A_z is the z-component of the magnetic vector potential. The vector of spatial coordinates is given as: $\vec{x} = (x, y) \in \mathbb{R}^2$. Equation (2) is solved by using the finite element method (FEM) with the inhouse code Niobe.

III. Karhunen-Loève expansion

The magnetic reluctivity is expressed via a finite number of random variables by the truncated KLE [1] as follows:

$$\nu_c(\vec{x}, \vec{\xi}(\theta)) \approx \overline{\nu}(\vec{x}) + \sum_{i=1}^{M} \sqrt{\lambda_i} f_i(\vec{x}) \xi_i(\theta), \tag{3}$$

where $\overline{\nu}$ is the mean value of the random field, $\vec{\xi}$ is a vector of mutually uncorrelated orthonormal random variables, f_i are orthonormal eigenfunctions and λ_i are eigenvalues. The eigenfunctions and the eigenvalues that appear in the KLE are obtained by solving the Fredholm integral equation:

$$\int_{D_c} \mathrm{Cov}(\vec{x}, \vec{y}) f_i(\vec{x}) \mathrm{d}\vec{x} = \lambda_i f_i(\vec{y}), \tag{4}$$

where Cov is the covariance function. By applying the Galerkin method the Fredholm integral equation results in a generalized eigenvalue problem,

$$\mathbf{A}\mathbf{f} = \lambda \mathbf{B}\mathbf{f}, \tag{5}$$

with matrices $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$ and eigenvector $\mathbf{f} \in \mathbb{R}^n$. Matrix $\mathbf{A}$ is symmetric and $\mathbf{B}$ is a diagonal matrix. The number of degrees of freedom is denoted as n. In this paper we assume a Gaussian covariance function given as,

$$\mathrm{Cov}(\vec{x}, \vec{y}) = \sigma^2 \exp\left(- \frac{||\vec{x} - \vec{y}||_{l_2}^2}{d^2} \right). \tag{6}$$

where σ and d are the standard deviation and the correlation length of the random field, respectively.

IV. Field quality

The radial component of the magnetic flux density is expanded as

$$B_r(r_0, \phi) = \sum_{k=1}^{\infty} (B_k(r_0) \sin(k\phi) + A_k(r_0) \cos(k\phi)), \tag{7}$$

where k is the pole-pair number, r_0 and ϕ are the radius of the reference circle, located at the center of the magnet, and the angle variable, respectively. The components B_k and A_k are known as the normal and the skew component, respectively. By using the complex-valued Fourier coefficients $\underline{c}_k$ of the magnetic vector potential A_z, evaluated at the reference circle, the multipole coefficients are computed as follows:

$$B_k = -\mathrm{Re}\{\underline{c}_k\} \frac{k}{r_0}, \qquad A_k = -\mathrm{Im}\{\underline{c}_k\} \frac{k}{r_0}. \tag{8}$$

Multipole Coefficients	Mean / T	Std / T
B_1	$5.459 \ 10^{-2}$	$3.057 \ 10^{-4}$
B_3	$-7.312 \ 10^{-5}$	$6.506 \ 10^{-7}$
B_5	$4.577 \ 10^{-6}$	$6.557 \ 10^{-8}$

TABLE I: Statistics for the multipole coefficients

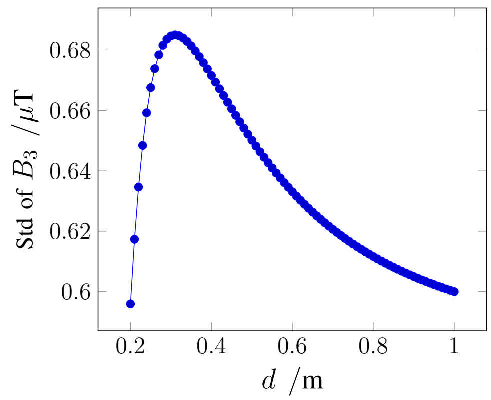

Fig. 2: The standard deviation of the 3rd multipole coefficient

In this paper the mean value and the standard deviation of the normal component B_k are computed by using the stochastic collocation method [6].

V. Results

The Fredholm integral equation (4) is solved for $\sigma = 5$, $d = 0.5$m. The number of turns in the switching coils is $N_{t1,2} = 56$ and the current is $I_{1,2} = 57$A. The steerer coil is not excited within this computations. The mean value of the random reluctivity is modeled as constant $\overline{\nu} = 795$ H^{-1}m and the random variables are uniformly distributed as $\xi_i \sim \mathcal{U}(-\sqrt{3}, \sqrt{3})$. Both, the air region and the coil region have the reluctivity of vacuum, $\nu_e = \nu_j = \mu_0^{-1}$. Table I shows the mean value and the standard deviation of the first, third and the fifth normal multipole coefficients. In Fig. (2) the standard deviation of the 3rd normal multipole, also known as sextopole, coefficient is depicted for varying correlation length d of the random field. The sextopole coefficient is most sensitive to uncertainties with a correlation length of approximately $d = 0.3$m. The standard deviation is decreasing for $d > 0.3$m.

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